

# SINGLE INPUT VARIABLE UNIVERSE FUZZY CONTROLLER WITH CONTRACTION-EXPANSION FACTOR FOR INVERTED PENDULUM IN REAL TIME

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**Abstract.** In Conventional Fuzzy Logic Controller (CFLC), input variables are the error ( $E$ ) and the change-in-error ( $EC$ ) regardless of the complexity of controlled plant. The rule table constructed for CFLC is two dimensional in input space. This two dimensional rule table can be reduced to a single dimension by the signed distance method. This method also reduces the number of rules significantly. In this paper, a fuzzy controller is designed with the signed distance input variable. Then to improve the performance of this controller the technique of variable universe of discourse is used. After using these two techniques simultaneous to design a fuzzy controller, it is implemented to stabilize inverted pendulum in real time.

## Keywords

*Contraction-expansion factor, inverted pendulum, single input fuzzy controller, variable universe.*

## 1. Introduction

The inverted pendulum is a multivariable, nonlinear fast reaction and unstable system [1]. The dynamic description of the inverted pendulum is slightly complicated. As it is a challenging problem to stabilize an inverted pendulum, therefore it can also be used to analyze the performance of any control method. Fuzzy control, variable structure control and robust control are some of the methods which commonly used to solve this problem. The performance of Fuzzy Logic Controller (FLC) depends on the number of its inference rules. The performance of the FLC can be easily enhanced by increasing the number of rules. But the large set of rules also requires more computational time [2]. This problem is solved by the introduction of a Single input Fuzzy Logic Controller (SFLC) [3]. In conventional fuzzy controllers, the input variables are mostly the error and

the change-in-error but in SFLC the input variable is the signed distance. This signed distance variable is sole fuzzy input variable in single input fuzzy logic controller.

Traditional fuzzy controller has many advantages, but its control accuracy is low [4]. So this type of method is not appropriate in such applications where highly precise control is required. For high precision, a variable universe adaptive fuzzy controller was proposed by professor Li in 1999 [5]. The controlling power of variable adaptive fuzzy control is verified for effective dealing with nonlinear system [6]. So in this paper a controller is designed using the technique of submission.

## 2. Modelling of Inverted Pendulum

### 2.1. Inverted Pendulum Structure

The structure of inverted pendulum is shown in Fig. 1. After ignoring the air resistance and other frictions, inverted pendulum can be simplified as a system of the cart and a quality rod, where  $M$  is the mass of cart,  $m$  is the mass of pendulum,  $l$  is the length of pendulum,  $\theta$  is the angle between pendulum and vertical and  $F$  is the external force acting on the system.

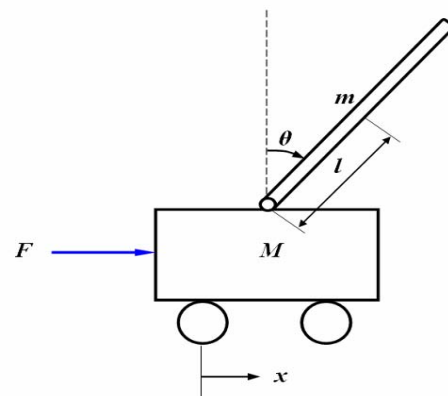


Fig. 1: Schematic diagram of inverted pendulum system.

Force  $F$  is imposed by a DC motor and this force makes the cart move around the rail. The main objective is to erect the stable pendulums mounted on the cart, within the limited rail length and to achieve dynamic balance.

### 2.2. Mathematical Model of Inverted Pendulum

The mathematical model of the inverted pendulum system is established using Lagrange equation, taking the state variables [13]:

$$x_1 = x, x_2 = \theta, x_3 = \dot{x}, x_4 = \dot{\theta}.$$

The equilibrium state is taken around

$$X = [x \ \theta \ \dot{x} \ \dot{\theta}]^T = [0 \ 0 \ 0 \ 0]^T.$$

Table 1 shows the parameters of the inverted pendulum used to derive state space model.

Tab.1: Parameters of inverted pendulum [13].

Parameter	Definition	Value	Unit
M	Mass of cart	1,096	kg
m	Mass of pendulum	0,125	kg
l	Length of pendulum	0,0775	m
g	Gravity constant	9,8	N/Kg

By substituting the parameters, the following linear model is obtained:

$$\begin{cases} \dot{x}(t) = Ax + Bu \\ y(t) = Cx + Du \end{cases} \quad (1)$$

where:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 29,4 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 3 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

### 3. Single Input Fuzzy Logic Controller

The Conventional Fuzzy Logic Controller (CFLC) has two inputs, which are mostly the error and the change-in-error. It requires a 2-dimensional rule table for inference. The rule table for CFLC with two inputs (error & change-in-error) is shown in Tab. 2. This rule table is in the skew-symmetric form. It can be observed from Tab. 2 that the output membership is same in a diagonal direction. Each point on the particular diagonal line has magnitude that is

proportional to the distance from its main diagonal line (LZ). For any combination of  $(e, \dot{e})$ , the output membership function will lie in any one of the diagonal line ( $L_{NB}, L_{NM}, L_{NS}, L_Z, L_{PS}, L_{PM}, L_{PB}$ ). The main diagonal line ( $L_Z$ ) can be a representation as [7]:

$$\dot{e} + \lambda e = 0. \quad (2)$$

Where,  $\lambda$  is the slope magnitude of the main diagonal line  $L_Z$ . The distance from any point  $(e, \dot{e})$  to the main diagonal line can be written as [7]:

$$d = \frac{\dot{e} + \lambda e}{\sqrt{1 + \lambda^2}}. \quad (3)$$

Depending on the distance  $d$ , the new rule table can be constructed and given in Tab.3. Rule table is one dimensional and contains only seven rules and confirms linear control surface. Number of input for FLC will be one and structure of single input FLC is given in Fig. 2 [8]. The calculated distance ( $d$ ) is the only input to the fuzzy logic controller.

Tab.2: CFLC rule base [2].

$\dot{e}$	NB	NM	NS	Z	PS	PM	PB
e	NB	NB	NB	NB	NM	NS	Z
NB	NB	NB	NB	NB	NM	NS	Z
NM	NB	NB	NB	NM	NS	Z	PS
NS	NB	NB	NM	NS	Z	PS	PM
Z	NB	NM	NS	Z	PS	PM	PB
PS	NM	NS	Z	PS	PM	PB	PB
NM	PM	NS	Z	PS	PM	PB	PB
NS	PB	Z	PS	PM	PB	PB	PB

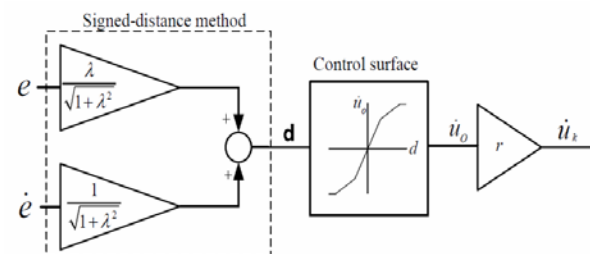


Fig. 2: Single input fuzzy logic controller.

Tab.3: SFLC rule base.

d	L <sub>NL</sub>	L <sub>NM</sub>	L <sub>NS</sub>	L <sub>Z</sub>	L <sub>PS</sub>	L <sub>PM</sub>	L <sub>PL</sub>
U	NL	NM	NS	Z	PS	PM	PL

### 4. Variable Universe Fuzzy Logic

Let  $X_i = [-E_i, E_i] (i = 1, 2, \dots, n)$  be the universe of the input variable  $x_i (i = 1, 2, \dots, n)$  and  $Y = [-U, U]$  be the universe of the output variable  $y$ .  $\phi_i = \{A_{ij}\}_{(j=1,2,\dots,m)}$  stand for the fuzzy sets  $X_i$  and  $\psi_j = \{B_j\}$  stand for the fuzzy sets  $Y$ .  $\phi_i$  and  $\psi_j$  can be called the linguistic variables. Fuzzy inference rule set  $\{R_s\}_{(s=1,2,\dots,z)}$  can be formed as:

$$R_S : \text{If } x_1 \text{ is } A_{1j}, \dots, \text{ and } x_n \text{ is } A_{nj} \text{ then } y = B_j. \quad (4)$$

The so-called variable universe means that some universes such as  $X_i$  and  $Y$ , can change along with changing variables  $x_i$  and  $y$  [9]. The transformed universe discourse is denoted as:

$$X_i(x_i) = [-\alpha_i(x_i)E_i, \alpha_i(x_i)E_i], \quad (5)$$

$$Y(y) = [-\beta(y)U, \beta(y)U]. \quad (6)$$

Where  $\alpha_i(x_i)$  and  $\beta(y)$  are contraction-expansion factors [10]. The varying universe is shown in Fig. 3 [11]:

For  $\alpha(x)$  be the contraction-expansion factor, following conditions should be satisfied [6]:

1. duality:  $(\forall x \in X) (\alpha(x) = \alpha(-x))$ ,
2. zero kept:  $\alpha(0) = 0$ ,
3. monotonicity:  $\alpha$  is strictly monotonically increasing on  $[0, E]$ ,
4. compatibility:  $(\forall x \in X) (|x| \leq \alpha(x)E)$ ,
5. normality:  $\alpha(\pm E) = 1$ .

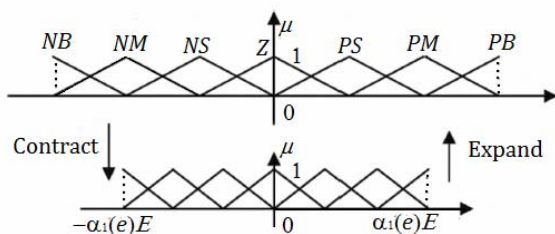


Fig. 3: Variable situation of the universe.

Therefore change rule of  $\alpha(x)$  is:

$$\Delta\alpha = k \cdot \Delta x \cdot x \cdot (1 - \alpha). \quad (7)$$

Where,  $k$  is a proportionality constant.

On moving  $\Delta x$  to left and assuming  $\Delta x \rightarrow 0$ , we will get;

$$\frac{d\alpha(x)}{dx} = kx(1 - \alpha(x)), \quad (8)$$

$$\frac{d\alpha(x)}{1 - \alpha(x)} = kx \cdot dx. \quad (9)$$

Integrating both sides to obtain  $\alpha(x)$

$$\alpha(x) = 1 - \eta e^{-kx^2}. \quad (10)$$

Assume  $\beta(t)$  is the universe contraction-expansion

factor of output  $Y$ .  $\beta(t)$  is designed with the principle of

weighted integral [4].

$$\beta(t) = K_t \int_0^t [e, ec] P_n d\tau + \beta(0). \quad (11)$$

Where  $K_t$  is a proportionality constant,  $P_n = [p_1, p_2]^T$  is a constant vector.

When  $\beta(t) = 1, K_t = 1, P_n = [1, 1]^T$ .

### 5. Control Scheme For Inverted Pendulum

There are two types of error for inverted pendulum, i.e. error in cart position ( $E_1$ ) and error in the angle of pendulum ( $E_2$ ). The derivatives of both of these errors will give velocity of cart ( $EC_1$ ) and angular velocity of pendulum ( $EC_2$ ). These four variables make the inverted pendulum a four dimensional system. In order to simplify the complexity of the system both errors ( $E_1$  &  $E_2$ ) and change-in-errors ( $EC_1$  &  $EC_2$ ) should be synthesize into only two variables the error ( $E$ ) and the change in error ( $EC$ ). This can be done by the help of Information Fusion Method [12]. After this, the signed distance variable ( $d$ ) is obtained by the help of Signed Distance Method [7]. This signed distance variable is fed to fuzzy controller as sole fuzzy input. Then variable universe technique is used to improve the accuracy and respond time of the system [5].

#### 5.1. Implementation of Information Fusion

Error  $E$  and change-in-error  $EC$  can be defined as:

$$E = [k_1 \quad k_2] \cdot \begin{bmatrix} x \\ \theta \end{bmatrix}, \quad (13)$$

$$EC = [k_4 \quad k_5] \cdot \begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix}. \quad (14)$$

where  $(k_1, k_2, k_3, k_4)$  are synthesis parameters.

Now a state feedback matrix for the state equation has to be designed. For this, make the quadratic performance index function [12];

$$J = \frac{1}{2} \int_0^{\infty} (X^T Q X + U^T R U) dt, \tag{15}$$

where the positive semi definite matrix  $Q = \text{diag} (1000, 0, 200, 0)$  and symmetric positive definite matrix  $R = 1$ .

For solving the Riccati equation:

$$ATP + PA - PBR - 1BTP + Q = 0. \tag{16}$$

The optimal feedback gain matrix values can be obtained:

$$K = R^{-1} B^T P. \tag{17}$$

$$K = (-31,623 \ -20,151 \ 72,718 \ 13,155)$$

Let  $\|K\|$  be;

$$\|K\| = \sqrt{[(K_1)^2 + (K_2)^2 + (K_3)^2 + (K_4)^2]}. \tag{18}$$

Now error  $E$  and error variation  $EC$  are:

$$E = [-0,38164 \quad 0,8776] \cdot \begin{bmatrix} x \\ \theta \end{bmatrix}, \tag{19}$$

$$EC = [-0,2432 \quad -0,15876] \cdot \begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix}. \tag{20}$$

### 5.2. The Signed Distance Variable

The error  $E$  and the change-in-error  $EC$  are combined to obtain the signed distance  $d$  by using (3). The gain factor for error is taken as 1,1 whereas for change-in-error it is taken as 1. The schematic diagram for SFLC is shown in Fig. 2.

### 5.3. Variable Universe Fuzzy Controller

The universe contraction-expansion factor of  $D_s$  from (10) is:

$$\alpha(e) = 1 - \eta e^{-ke^2}.$$

On choosing,  $\eta = 0,27, k = 10^{-2}$ ;

$$\alpha(e) = 1 - 0,27e^{-10^{-2}e^2}.$$

Assume  $\beta(t)$  is the universe contraction-expansion factor of output  $U$ . Then  $\beta(t)$  from (12) is:

$$\beta(t) = \int_0^t d \, d\tau + 1.$$

## 6. Simulation Results

MATLAB SIMULINK is used in this paper for simulation of the controller to control inverted pendulum. Then this controller is implemented on Googol 1-stage linear inverted pendulum in real time. The initial fuzzy universe of  $D_s$  is taken  $[-1 \ 1]$  and for the output  $U$  it is  $[-1 \ 1]$ . Mamdani's fuzzy inference method is used in the controller's fuzzifier and defuzzifier. The membership functions of input and output variables contain seven variables and shown in Fig. 4. The control rules designed for inverted pendulum are described in Tab. 4.

Tab.4: Fuzzy control rule table.

$D_s$	LNB	LNM	LNS	LZ	LPS	LPM	LPB
U	NB	NM	NS	Z	PS	PM	PB

For defuzzification, method of centroid calculation is used which returns the center of area under the curve.

$$\text{Centroid} = \frac{\int u \mu_u(u) du}{\int \mu_u(u) du}.$$

In control of inverted pendulum, the stability of inverted pendulum at the given position is highly sensitive to the initial position of the cart and the initial angle of inverted pendulum. So first, simulation results are discussed and then real time results are discussed. Now following two cases are taken with different initial conditions for simulation:

**Case A:** In this case the initial simulation conditions are set at:  $x = 0,1 \text{ m}, \theta_1 = 0,1 \text{ rad}$ .

The length of simulation step is taken 1 ms and simulation time is 5 seconds. Now the cart is required to move at  $x = 0$ . Simulation results for *case A* are shown in Fig. 6. From the simulation results it can be observed that system reach equilibrium position within 2 seconds.

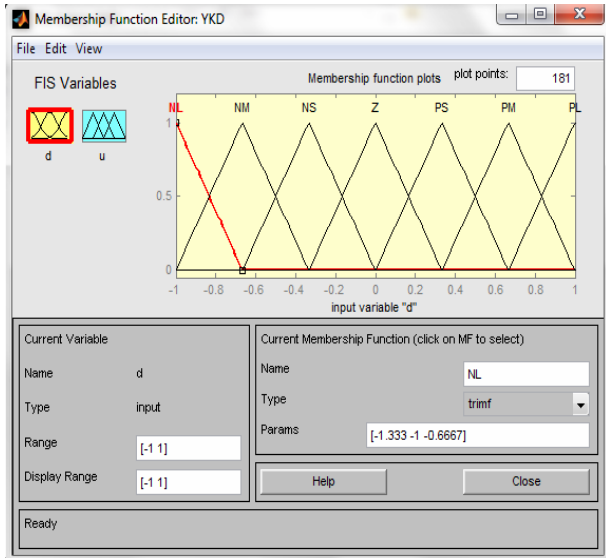


Fig. 4: The membership functions of input  $D_s$ .

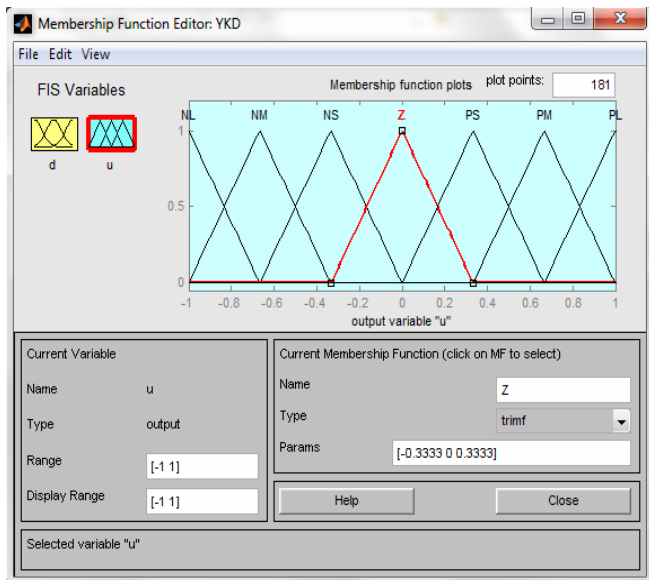


Fig. 5: The membership functions of output  $U$ .

**Case B:** In case B the initial simulation conditions are set at:  $x = 0,1$  m,  $\theta_1 = -0,1$  rad.

The length of simulation step is taken 1ms and simulation time is 5 seconds. Now the cart is required to move at  $\Delta x = 0$ . Simulation results for case B are shown in Fig. 7. From the simulation results it can be observed that system reach equilibrium position within 2,5 seconds.

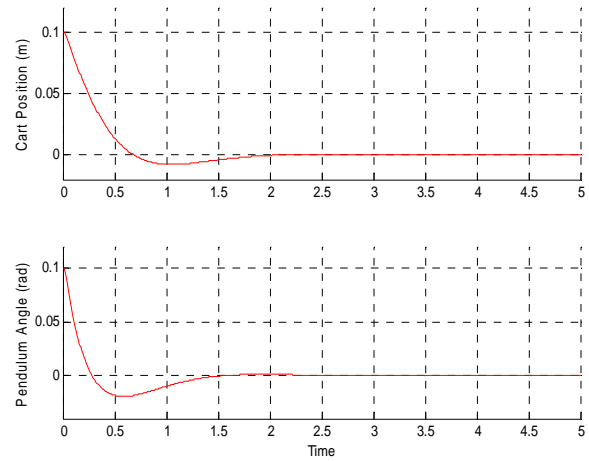


Fig. 6: Simulation results for case A.

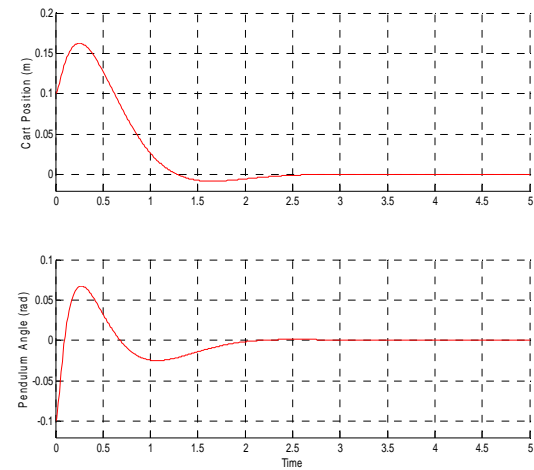


Fig. 7: Simulation results for case B.

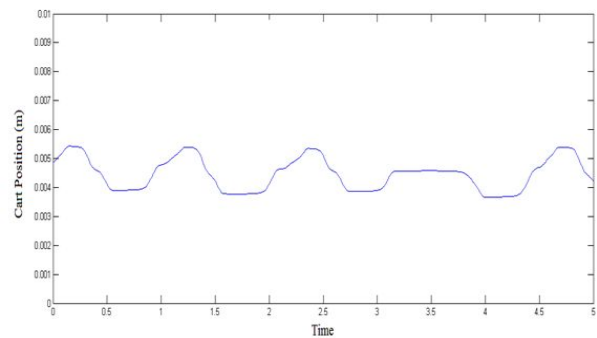


Fig. 8: Cart position in real time.

The curve for cart position and pendulum angle for real time is shown in Fig. 8 and Fig. 9. Here the main objective is to stabilize the angle. The disturbance in position is only about 0,0015 m. Figure 9 shows the pendulum angle which is stabilized around 3,1415 rad (180 degrees). The angle of pendulum varies between the ranges from 3,14 rad to 3,15 rad.

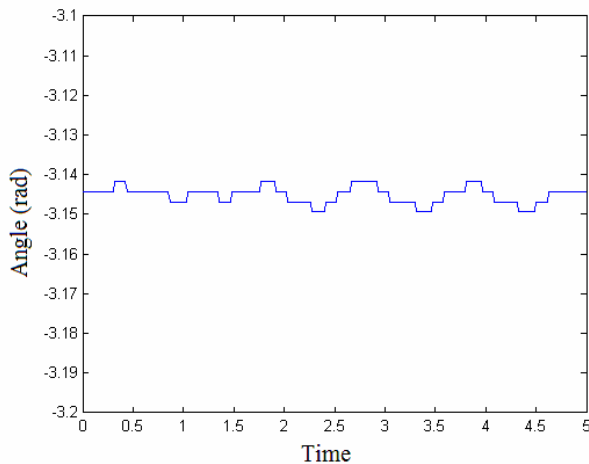


Fig. 9: Pendulum angle in real time.

## 7. Conclusion

An inverted pendulum system in real time is taken as controlled object for stabilization. To simplify the controller design, two pairs of the error and the change-in-error are combined into single pair of the error (E) and the change-in-error (EC) by the method of information fusion. Then, E and EC are merged to form the signed distance (d) variable with the help of signed distance method. A suitable single input fuzzy controller with variable universe of discourse is designed and the length of the universe of discourse is adjusted by universe contraction-expansion factor.

Simulation results are obtained for two different cases with different initial conditions. Then this controller is implemented on inverted pendulum in real time and curves for position and angle are obtained. From the real time results, it can be observed that performance of the controller is precise in nature and also poses high degree of accuracy to the conventional fuzzy controller.

## References

- [1] QING-RUI, Li, Tao WEN-HUA, Na SUN, Zhang CHONG-YANG and Yao LING-HONG. Stabilization Control of Double Inverted Pendulum System. In: *Proc. of 3rd International Conference on Innovative Computing Information and Control, 2008. ICICIC '08*. Dalian: IEEE, 2008, pp. 417-420. ISBN 978-0-7695-3161-8. DOI: 10.1109/ICICIC.2008.662.
- [2] AMJAD, M., M. I. KASHIF, S. S. ABDULLAH and Z. SHAREEF. A simplified intelligent controller for ball and beam system. In: *Proc. of 2nd International Conference on Education Technology and Computer (ICETC10)*. Shanghai: IEEE, 2010, pp. 494-498. ISBN 978-1-4244-6367-1. DOI: 10.1109/ICETC.2010.5529491.
- [3] BYUNG-JAE, Choi, Kwak SEONG-WOO and Kim BYUNG KOOK. Design of a single-input fuzzy logic controller and its properties. *Fuzzy Sets and Systems*. 1999, vol. 106, iss. 3, pp. 299-308. ISSN 0165-0114. DOI: 10.1016/S0165-

0114(97)00283-2.

- [4] BEIBEI, Hou and Yan GAO. Variable Universe Fuzzy Controller with Correction Factors for Ball and Beam System. In: *Proc. of 3rd International Workshop on Intelligent Systems and Applications (ISA), 2011*. Wuhan: IEEE, 2011, pp. 1-4. ISBN 978-1-4244-9855-0. DOI: 10.1109/ISA.2011.5873283.
- [5] HONGXING, Li. Adaptive fuzzy controllers based on variable universe. *Science in China: Series E: Technological Sciences*. 1999, vol. 42, no. 1, pp. 10-20. ISSN 1006-9321. DOI: 10.1007/BF02917053.
- [6] XIAOFENG, Gong, Li HONGXING, Deng GUANNAN and Guo HAIGANG. Variable Universe Adaptive Fuzzy Control on the Triple Inverted Pendulum and Choosing Contraction-Expansion Factor. In: *Proc. of International Conference on Artificial Intelligence and Computational Intelligence, 2009. AICI '09*. Shanghai: IEEE, 2009, pp. 63-67. ISBN 978-1-4244-3835-8. DOI: 10.1109/AICI.2009.245.
- [7] BYUNG-JAE, Choi, Kwak SEONG-WOO and Kim BYUNG KOOK. Design and Stability Analysis of Single-Input Fuzzy Logic Controller. *IEEE Transactions on Systems, Man, and Cybernetics: Part B: Cybernetics*. 2000, vol. 30, no. 2, pp. 303-309. ISSN 1083-4419. DOI: 10.1109/3477.836378.
- [8] TAEED, F., Z. SALAM and S. M. AYOB. Implementation of Single Input Fuzzy Logic Controller for Boost DC to DC power converter. In: *Proc. of IEEE International Conference on Power and Energy (PECon2010)*. Kuala Lumpur: IEEE, 2010, pp. 797-802. ISBN 978-1-4244-8947-3. DOI: 10.1109/PECON.2010.5697688.
- [9] CAO, Dan-yang, Shui-ping ZENG and Jin-hong LI. Variable universe fuzzy expert system for aluminum electrolysis. *Transactions of Nonferrous Metals Society of China*. 2011, vol. 21, iss. 2, pp. 429-436. ISSN 1003-6326. DOI: 10.1016/S1003-6326(11)60732-2.
- [10] JIANG, Wang, Qiao GUO-DONG and Deng BIN.  $H_\infty$  Variable universe adaptive fuzzy control for chaotic system. *Chaos, Solitons & Fractals*. 2005, vol. 24, iss. 4, pp. 1075-1086. ISSN 0960-0779. DOI: 10.1016/j.chaos.2004.09.056.
- [11] JING, Wang and Zhang WEI-WEI. Chaos control via variable universe fuzzy theory in auto gauge control system. In: *Proc. of International Conference on Computer Application and System Modeling (ICCASM2010)*. Taiyuan: IEEE, 2010, pp. V1-363-V1-369. ISBN 978-1-4244-7235-2. DOI: 10.1109/ICCASM.2010.5619264.
- [12] WANG, Lin, Shifu ZHENG, Xiping WANG and Liping FAN. Fuzzy control of a double inverted pendulum based on information fusion. In: *Proc. of International Conference on Intelligent Control and Information Processing (ICICIP2010)*. Dalian: IEEE, 2010, pp. 327-331. ISBN 978-1-4244-7047-1. DOI: 10.1109/ICICIP.2010.5564318.
- [13] Inverted pendulum: Experiment Manual. In: *Googol Technology* [online]. 2nd, 2006. Available at: <http://www.googoltech.com/uploads/catalog/569/GLIP%20experiment%20and%20instruction%20manual.pdf>.

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